Test Kaleidoscope Mathematics (Dimensional Analysis), October 8th 2018

All answers need to be justified. Each exercise has a certain amount of points, summing up 9 'points. The grade will be computed as *grade* = 1 + (*points obtained*)/9. Please answer each exercise on a different sheet to facilitate the grading. A table with dimensions can be found on the second page.

Exercise 1 (6 points)

The velocity v at which a pressure wave is traveling through an elastic pipe depends on: the radius R of the pipe, the thickness of the pipe's wall h, the mass density of the pipe's material ρ, and the elastic modulus of the pipe's material E.

(a) $\boxed{2.0}$ Assume first that v does not depend on h. Using dimensional analysis, build a mathematical model for v in terms of R, ρ and E.

(b) $\boxed{3.0}$ Using dimensional analysis, build a mathematical model for v in terms of h, R, ρ and E.

(c) $\boxed{1.0}$ Experiments show that v is proportional to the square root of h. Using that knowledge, obtain an explicit form for the mathematical model obtained in Exercise 1(b).

Exercise 2 (3 points)

The displacement $x(t)$ of the roof of a building during an earthquake satisfies the following differential equation:

$$x''(t) + \omega^2 x(t) = -y$$

with t the time, m the mass of the building, ω the main vibration frequency of the building, and y the acceleration of the ground.

(a) $\boxed{1.5}$ Using dimensional analysis, show that

$$x = \frac{y}{\omega^2} g(\Pi),$$

with $g(\Pi)$ a general univariate function. Determine Π in terms of ω, y and/or t.

(b) $\boxed{1.5}$ Find the differential equation satisfied by $g(\Pi)$. Assume that y does not depend on time.

Acceleration	LT^{-2}	Enthalpy	ML^2T^{-2}
Angle	1	Entropy	$ML^2T^{-2}\theta^{-1}$
Angular Acceleration	T^{-2}	Gas Constant	$L^2T^{-2}\theta^{-1}$
Angular Momentum	ML^2T^{-1}	Internal Energy	ML^2T^{-2}
Angular Velocity	T^{-1}	Specific Heat	$L^2T^{-2}\theta^{-1}$
Area	L^2	Temperature	θ
Energy, Work	ML^2T^{-2}	Thermal Conductivity	$MLT^{-3}\theta^{-1}$
Force	MLT^{-2}	Thermal Diffusivity	L^2T^{-1}
Frequency	T^{-1}	Heat Transfer Coefficient	$MT^{-3}\theta^{-1}$
Concentration	L^{-3}		
Length	L	Capacitance	$M^{-1}L^{-2}T^4I^2$
Mass	M	Charge	TI
Mass Density	ML^{-3}	Charge Density	$L^{-3}TI$
Momentum	MLT^{-1}	Conductivity	$M^{-1}L^{-3}T^3I^2$
Power	ML^2T^{-3}	Electric Current Density	$L^{-2}I$
Pressure, Stress, Elastic Modulus	$ML^{-1}T^{-2}$	Electric Current	I
Surface Tension	MT^{-2}	Electric Displacement	$L^{-2}TI$
Time	T	Electric Potential	$ML^2T^{-3}I^{-1}$
Torque	ML^2T^{-2}	Electric Field Intensity	$MLT^{-3}I^{-1}$
Velocity	LT^{-1}	Inductance	$ML^2T^{-2}I^{-2}$
Viscosity (Dynamic)	$ML^{-1}T^{-1}$	Magnetic Field Intensity	$L^{-1}I$
Viscosity (Kinematic)	L^2T^{-1}	Magnetic flux	$L^2MT^{-2}I^{-1}$
Volume	L^3	Permeability	$MLT^{-2}I^{-2}$
Wave Length	L	Permittivity	$M^{-1}L^{-3}T^4I^2$
Strain	1	Electric Resistance	$ML^2T^{-3}I^{-2}$

Table 1.1 Fundamental dimensions for commonly occurring quantities. A quantity with a one in the dimensions column is dimensionless.